Logs. (Logarithms)

The log of a number is the power to which the base number must be raised to get that number

If: $a = b^c$; then $\boxed{\log_b a = c}$ to convert back into index form then move b and c

eg: $32 = 2^5$; $\log_2 32 = 5$

$\boxed{\log_{\boxed{5}} \boxed{125}} = 3$ read: 'log of 125 to the base 5 = 3'

base number

The log of 125 to the base 5 =

'How many times do I multiply 5 (base) by itself to get 125' = 3

eg $a = 4, 125$
$b = 2, 5$
$c = 2, 3$

as $4 = 2 \times 2$ or $125 = 5 \times 5 \times 5$
ie $4 = 2^2$ ie $125 = 5^3$

to find x, convert to log form

$200 = 2^x = \log_2 200 = 7.6$

Rules of logs:

1. $\log_a x \times y = \log_a x + \log_a y$

$\log_{10} 3 + \log_{10} 4 = \log_{10}(3\times 4) = \log_{10} 12 = 1.08$

i.e. if same base number and adding two logs then multiply numbers

eg $\log_2 5 + \log_2 6 = \log_2(5\times 6)$
$= \log_2 30$

or

$\log_2 x + \log_2 14 = \log_2(14x)$

2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

If log is a fraction, then both can be subtracted, provided base number is the same (and vice versa)

eg $\log_3 7 - \log_3 2 = \log_3\left(\frac{7}{2}\right)$

$\log_{10} 54 - \log_{10} 27 = \log_{10}\left(\frac{54}{27}\right) = \log_{10} 2$

③ $\log_a x^n = n \times \log_a x$

eg $\log_2 10^2 = 2 \times \log_2 10$

or $\log_3 27 = \log_3 3^3$

$= 3 \log_3 3$

Very important use of logs.

If power is an unknown.

here to power can be move to the front and multiplied

eg: $3^n = 1{,}000$ solve for n?

take the log of both sides to base 3

(can use any base but 3 is best (see rule 4))

$\Rightarrow \log_3 3^n = \log_3 1{,}000$ move n to front

$n \times \log_3 3 = \log_3 1000$ $\boxed{\log_3 3 = 1}$

$\Rightarrow n = \log_3 1000 = 6.3$

④ $\boxed{\log_a a = 1}$

$\log_a a = ?$ $a = a^?$ i.e. $\log_7 7 = x$

$$\log_a a^2 = ?$$

$$= 2 \log_a a$$

$$= 2 \times 1$$

$$= 2$$

e.g. $2^x = 1,000,000$

$\log_2 2^x = \log_2 1,000,000$

$x \times \log_2 2 = \log_2 1,000,000$

$x = \log_2 1,000,000$

$x = 19.93$ like example above.

⑤ $\log_a 1 = 0$

common log = $\log_{10}$

$$\boxed{\text{natural log} = \log_e = \ln}$$

e is an irrational number = 2.718 . . . . .

change of base rule

$$\log_b x = \frac{\log_a x}{\log_a b}$$

eg $\log_4 64 = \dfrac{\log_2 64}{\log_2 4} = 3$

$$= \frac{\log_2 2^6}{\log_2 2^2} = \frac{6 \times \log_2 2}{2 \times \log_2 2}$$

$$= \frac{6}{2} = 3$$

QUESTIONS

$$\log_2 x + 4\boxed{\log_x 2} = 5$$

$$\frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$$

$$\log_2 x + 4\left(\frac{1}{\log_2 x}\right) = 5$$

Introduce a new unknown. Let $\boxed{\log_2 x = y}$

$$\therefore \quad y + \frac{4}{y} = 5 \quad \ldots \text{(multiply by } y\text{)}$$

$$y^2 + 4 = 5y$$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1) = 0$$

$$\boxed{y = 4} \text{ or } \boxed{y = 1}$$

$$\log_2 x = 4 \quad \text{or} \quad \log_2 x = 1$$

$$x = 2^4 \qquad x = 2^1$$

$$\boxed{x = 16} \qquad \boxed{x = 2}$$

Log equations.

Get a single log in the equation and change to index

$$\log_2 x = 3 - \log_2(x-2)$$

$$\log_2 x + \log_2(x-2) = 3$$

$$\log_2 [x(x-2)] = 3$$ ... convert to index form

$$a = b^c$$

$$\log_b a = c$$

$$x(x-2) = 2^3$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$x = 4$ or $\boxed{\cancel{x = -2}}$

$$\boxed{\log_2(-2) = x}$$

$$-2 = 2^x$$

$$2^{-1} = \frac{1}{2^1}$$

$$-2 = 2^{x}$$

$\log(-a)$ is not defined. You can NEVER get the log of a negative number

$$\log_{10}(x^2+24) - \log_{10} x = 1$$

$$\log_{10}\left(\frac{x^2+24}{x}\right) = 1$$

$$\frac{x^2+24}{x} = 10^1 \quad \ldots (\times x)$$

$$x^2 + 24 = 10x$$

$$x^2 - 10x + 24 = 0$$

$$(x-6)(x-4) = 0$$

$$x = 6 \quad \text{or} \quad x = 4$$

$$\log_5 x = 1 + \boxed{\log_5\left(\frac{3}{2x-1}\right)}$$

$$\log_5 x - \log_5\left(\frac{3}{2x-1}\right) = 1$$

$$\log_5\left(x \div \frac{3}{2x-1}\right) = 1$$

$$\log_5\left(x \times \frac{(2x-1)}{3}\right) = 1$$

$$\log_5\left(\frac{2x^2-x}{3}\right) = 1$$

$$\frac{2x^2-x}{3} = 5^1$$

$$2x^2 - x = 15$$

$$2x^2 - x - 15 = 0$$

$$(2x+5)(x-3) = 0$$

(2x + 5)
(x − 3)

$$\boxed{x = -\frac{5}{2}} \quad x = 3$$